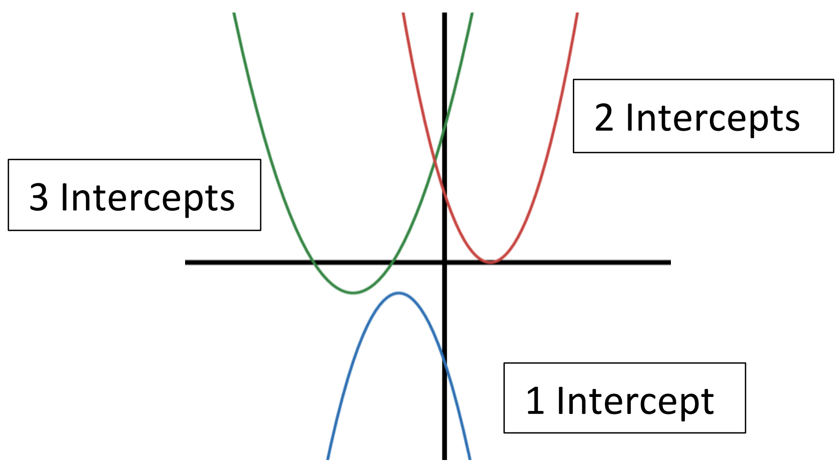
INTERCEPTS OF A PARABOLA – FACTORABLE

Most LINES have just two intercepts (or just one intercept if it's a horizontal or vertical line that's not one of the axes, or even infinitely many intercepts if the line is one of the axes.) But a PARABOLA may have one intercept, perhaps two intercepts, or possibly three intercepts:



It is impossible for a parabola to have NO intercepts, mainly because in a parabola opening up or down, the x-value can be any real number, meaning that the graph goes infinitely to the right and infinitely to the left. If follows that the parabola MUST pass through the y-axis somewhere, thus providing at least one intercept.

As in the chapter Graphing Parabolas, we're assuming parabolas that open up or down.

❑ *How Do We Find the Intercepts of a Parabola?*

Remember the method for finding the intercepts of a line? Well, an intercept is an intercept, so the rules for finding the intercepts of a parabola (or any graph at all!) are identical to the rules we learned before:

> x-intercepts are found by setting $\boldsymbol{y}$ **to 0**.
>
> y-intercepts are found by setting $\boldsymbol{x}$ **to 0**.

Also recall that an intercept is a point in the plane, and should be written as an ordered pair like (2, 0) or (0, –3). Ask your instructor if you need to write your intercepts this way.

EXAMPLE 1: **Find the intercepts of $y = x^2 - x - 6$.**

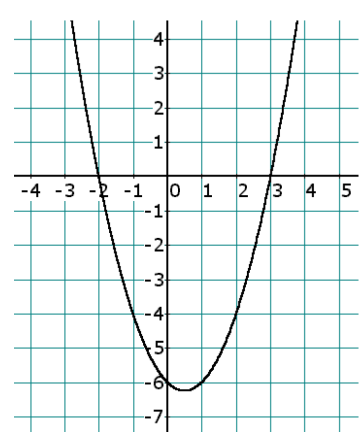
Solution:

x-intercepts: Setting $y = 0$

$$\Rightarrow \mathbf{0} = x^2 - x - 6$$
$$\Rightarrow 0 = (x+2)(x-3)$$
$$\Rightarrow x + 2 = 0 \text{ or } x - 3 = 0$$
$$\Rightarrow x = -2 \text{ or } x = 3$$

We conclude that

$\boxed{\text{the } x\text{-intercepts are } (-2, 0) \text{ and } (3, 0)}$



y-intercepts: Setting $x = 0$

$$\Rightarrow y = \mathbf{0}^2 - 6(\mathbf{0}) - 6 = -6$$

Therefore,

$\boxed{\text{the } y\text{-intercept is } (0, -6)}$

EXAMPLE 2: **Find the intercepts of $y = x^2 - 6x + 9$.**

Solution:

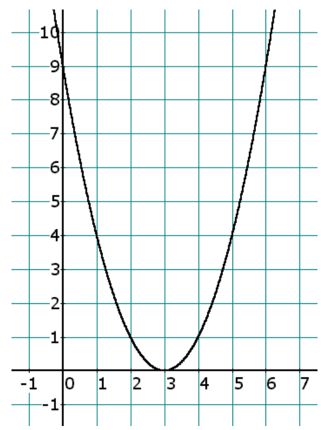
x-intercepts: Setting $y = 0$

$$\Rightarrow \mathbf{0} = x^2 - 6x + 9$$
$$\Rightarrow 0 = (x-3)(x-3)$$
$$\Rightarrow x - 3 = 0$$
$$\Rightarrow x = 3,$$

only one solution for x.

We conclude that

the x-intercept is $(3, 0)$



y-intercepts: Setting $x = 0$

$$\Rightarrow y = (\mathbf{0})^2 - 6(\mathbf{0}) + 9 = 9$$

Therefore,

the y-intercept is $(0, 9)$

Homework

Find all the **intercepts** of each parabola:

1. $y = x^2 - 25$
2. $y = 2x^2 - 3x - 35$
3. $y = 9x^2 + 6x + 1$
4. $y = -49x^2 + 28x - 4$
5. $y = 9x^2 - 1$
6. $y = -14x^2 + x + 3$
7. $y = 10x^2 + 27x + 14$
8. $y = 15x^2 - 34x + 15$

Solutions

1. (5, 0) (–5, 0) (0, –25)

2. (5, 0) $\left(-\frac{7}{2}, 0\right)$ (0, –35)

3. $\left(-\frac{1}{3}, 0\right)$ (0, 1)

4. $\left(\frac{2}{7}, 0\right)$ (0, –4)

5. $\left(\frac{1}{3}, 0\right)$ $\left(-\frac{1}{3}, 0\right)$ (0, –1)

6. $\left(-\frac{3}{7}, 0\right)$ $\left(\frac{1}{2}, 0\right)$ (0, 3)

7. $\left(-\frac{7}{10}, 0\right)$ (–2, 0) (0, 14)

8. $\left(\frac{5}{3}, 0\right)$ $\left(\frac{3}{5}, 0\right)$ (0, 15)

“Any fool can know. The point is to understand.”

– ***Albert Einstein***